



# The Gamma Ray Bursts Hubble diagram

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**Abstract.** Thanks to their enormous energy release, Gamma Rays Bursts (GRBs) have recently attracted a lot of interest to probe the Hubble diagram (HD) deep into the matter dominated era and hence complement Type Ia Supernovae (SNeIa). We consider here three different calibration methods based on the use of a fiducial  $\Lambda$ CDM model, on cosmographic parameters and on the local regression on SNeIa to calibrate the scaling relations proposed as an equivalent to the Phillips law to standardize GRBs finding any significant dependence. We then investigate the evolution of these parameters with the redshift to obtain any statistical improvement. Under this assumption, we then consider possible systematics effects on the HDs introduced by the calibration method, the averaging procedure and the homogeneity of the sample arguing against any significant bias.

## 1. Introduction

The observational evidences accumulated in the last years, from the anisotropy and polarization spectra of the cosmic microwave background radiation (CMBR), the large scale structure traced by galaxy redshift surveys, the matter power spectrum with the imprints of the Baryonic Acoustic Oscillations (BAO) and the Hubble diagram of SNeIa, definitely support the cosmological picture of a spatially

flat universe with a subcritical matter content ( $\Omega_M \sim 0.3$ ) and undergoing a phase of accelerated expansion. From a theoretical point of view, the problem today is the presence of too many ideas, ranging from the classical cosmological constant to scalar fields and higher order gravity theories all of them being more or less able to fit the available data. As often in science, adding further data and pushing the observed Hubble diagram to higher redshift, calling into cause the so energetic Gamma Ray Burst (GRBs), is the best strat-

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egy to put order in this theoretical scenario. We believe that the existence of many observationally motivated correlations, e.g. (Amati et al. 2008), offers the intriguing possibility of turning GRBs into standardizable candles just as SNeIa. Two main problems are actually still to be fully addressed. First, all the correlations have to be calibrated assuming a fiducial cosmological model to estimate the redshift dependent quantities. As a consequence, the so called circularity problem comes out and we try to investigate if the different strategies proposed to break it are viable solutions. On the other hand, there is up to now no any definitive understanding of the GRBs scaling relations so that one cannot anticipate whether the calibration parameters are redshift dependent. We address this question in a phenomenological way adopting different parameterizations.

## 2. GRBs scaling relations

To start, let us consider first the general case of two observable quantities  $(x, y)$  related by a power-law relation which, in a log-log plane, reads

$$\log y = a \log x + b . \quad (1)$$

Calibrating such a relation means determining the slope  $a$ , the zeropoint  $b$  and the scatter  $\sigma_{int}$  of the points around the best fit relation. Setting  $y = \kappa d_L^2(z)$  with  $\kappa$  a directly measurable redshift independent quantity and  $d_L(z)$  the luminosity distance, one can then estimate the distance modulus as :

$$\begin{aligned} \mu(z) &= 25 + 5 \log d_L(z) \\ &= 25 + (5/2)(a \log x + b - \log \kappa) \end{aligned} \quad (2)$$

In order to perform such an estimate, one has to select a sample of low redshift ( $z \leq 0.01$ ) objects with known distance and fit the scaling relation to infer the calibration parameters  $(a, b, \sigma_{int})$ . Then, one has to assume that such calibration parameters do not change with the redshift so that a measurement of  $(x, \kappa, z)$  and the use of the above scaling relation are sufficient to infer the distance modulus. This approach, in principle, can be adopted for long and short GRBs (Capozziello et al. 2011).

## 2.1. 2D empirical correlations

We limit here our attention only to two dimensional (hereafter, 2D) correlations since they can be investigated relying on a larger number of GRBs. These involve a wide range of GRBs properties related to both the energy spectrum and the light curve which are correlated with the isotropic luminosity  $L$  or the emitted collimation corrected energy  $E_\gamma$ . These last quantities depend on the luminosity distance  $d_L(z)$  as shown below:

$$L = 4\pi d_L^2(z) P_{bolo} , \quad (3)$$

$$E_\gamma = 4\pi d_L^2(z) S_{bolo} F_{beam} (1+z)^{-1} , \quad (4)$$

where  $P_{bolo}$  and  $S_{bolo}$  are the bolometric peak flux and fluence, respectively, while  $F_{beam} = 1 - \cos(\theta_{jet})$  is the beaming factor with  $\theta_{jet}$  the rest frame time of achromatic break in the afterglow light curve. The combination of  $x$  and  $y$  gives rise to the different GRBs correlations we will consider, namely the  $E_\gamma - E_{peak}$  (Ghirlanda et al. 2004), the  $L - E_{peak}$  (Schaefer 2003),  $L - \tau_{lag}$  (Norris et al. 2000),  $L - \tau_{RT}$  (Schaefer 2007) and  $L - V$  (Fenimore & Ramirez-Ruiz 2000).

## 2.2. Bayesian fitting procedure

Eq.(1) is a linear relation which can be fitted to a given dataset  $(x_i, y_i)$  in order to determine the two calibration parameters  $(a, b)$ . The above linear relations will be affected by an intrinsic scatter  $\sigma_{int}$  which has to be determined together with the calibration coefficients. To this aim, in the following we will resort to a Bayesian motivated technique (D'Agostini 2005), which, however, does not tell us whether this model fits well or not the data.

In order to sample the parameter space, we use a Markov Chain Monte Carlo (MCMC) method running two parallel chains and using the Gelman-Rubin (1992) test to check convergence (Cardone et al. 2011).

## 2.3. GRBs luminosity distances

A preliminary step in the analysis of the 2D correlations is the determination of the lumi-

luminosity  $L$  or the collimated energy  $E_\gamma$  entering as  $Y$  variable in the  $X$ - $Y$  scaling laws with  $(X, Y) = (\log x, \log y)$ . As shown in Eqs.(3)-(4), one has to determine the GRBs luminosity distance over a redshift range where the linear Hubble law does not hold anymore. Different strategies have been developed to tackle this problem. The simplest one is to assume a fiducial cosmological model and determine its parameters by fitting, e.g., the SNeIa Hubble diagram. The  $\Lambda$ CDM is usually adopted as fiducial model thus setting :

$$E^2(z) = \Omega_M(1+z)^3 + \Omega_\Lambda \quad (5)$$

with  $\Omega_\Lambda = 1 - \Omega_M$  because of the spatial flatness assumption. We determine the parameters  $(\Omega_M, h)$ , using the Union2 SNeIa sample (Kowalski et al. 2008) to get  $(\mu_i^{obs}, \sigma_{\mu_i})$  for  $N_{SNeIa} = 557$  objects over the redshift range  $(0.015, 1.4)$  and set  $(\omega_M^{obs}, \sigma_{\omega_M}) = (0.1356, 0.0034)$  for the matter physical density  $\omega_M = \Omega_M h^2$  and  $(h, \sigma_h) = (0.742, 0.036)$  for the Hubble constant. The best fit values turn out to be  $(\Omega_M, h) = (0.261, 0.722)$ .

Although the  $\Lambda$ CDM model fits remarkably well the data, it is nevertheless worth stressing that a different cosmological model would give different values for  $d_L(z)$  thus impacting the estimate of the calibration parameters  $(a, \sigma_{int})$ . Looking for a model independent approaches, we first resort to cosmography, i.e., we expand the scale factor  $a(t)$  to the fifth order and then consider the luminosity distance as a function of the cosmographic parameters (Capozziello & Izzo 2008; Izzo et al 2009).

As a further step towards a fully model independent estimate of the GRBs luminosity distances, one can use SNeIa as distance indicator based on the naive observations that a GRBs at redshift  $z$  must have the same distance modulus of SNeIa having the same redshift (Capozziello & Izzo 2008). Interpolating the SNeIa Hubble diagram gives the value of  $\mu(z)$  for a subset of the GRBs sample with  $z \leq 1.4$  which can then be used to calibrate the 2D correlations (Kodama et al. 2008). Assuming that this calibration is redshift independent, one can build up the Hubble diagram at higher redshifts using the calibrated correlations for the

remaining GRBs in the sample. As in Cardone et al 2009, we have used an approach based on the local regression technique which combines much of the simplicity of linear least squares regression with the flexibility of nonlinear regression.

### 3. Calibration parameters

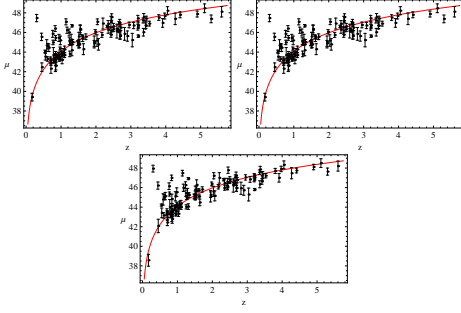
While the  $X$  quantities are directly observed for each GRB, the determination of  $Y$  (either the luminosity  $L$  or the collimated energy  $E_\gamma$ ) needs for the object's luminosity distance. The three methods described above allows us to get three different values for  $Y$  so that it is worth investigating whether this has any significant impact on the calibration parameters  $(a, b, \sigma_{int})$  for the correlations of interest. We will refer hereafter to the three samples with the  $Y$  quantities estimated using the luminosity distance from the fiducial  $\Lambda$ CDM cosmological model, the cosmographic parameters and the local regression method as the  $F$ ,  $C$  and  $LR$  samples, respectively. As a general result, we find that the fit is always quite good, with reduced  $\chi^2$  values close to 1, in all the cases independently of the 2D correlation considered and the distance estimate method adopted.

The best fit coefficients and the median values clearly show that the calibration based on the fiducial  $\Lambda$ CDM model leads to steeper scaling laws for the most of cases. On the contrary, shallower slopes are obtained using the  $C$  or  $LR$  samples with the  $L$ - $V$  relation as unique exception. Although the differences in the slopes are not statistically meaningful because of the large uncertainties, we find that the change in the slope is not induced by the different luminosity distances adopted.

### 4. Evolution with redshift

It is not clear whether the calibration parameters  $(a, b, \sigma_{int})$  evolve with redshift or not. To investigate this issue, we consider two different possibilities for the evolution with  $z$ . First, we consider the possibility that the slope is constant, but the zeropoint is evolving. In particular, we assume :

$$y = B(1+z)^\alpha x^A \longrightarrow Y = \alpha \log(1+z) + aX + b \quad (6)$$



**Fig. 1.** GRBs Hubble diagrams (HDs) averaging over the six 2D correlations. Three panels refer to the HDs derived using the calibration based on the fiducial  $\Lambda$ CDM model (left), the cosmographic parameters (right), and the local regression (down).

with  $(X, Y) = (\log x, \log y)$  and  $(a, b) = (A, \log b)$ . Comparing the previous constraints, we note that both the best fit and median values of the slope parameter  $a$  are significantly shallower than in the no evolution case. However, the 68% confidence ranges typically overlap quite well so that, from a statistical point of view, such a result should not be overrated. As such, we consider a most conservative option to assume that the GRBs scaling relations explored here do not evolve with  $z$ .

As an alternative parametrization, we allow for an evolution of the slope and not only the zeropoint of the 2D correlations. We fit the data using :

$$Y = (a_0 + a_1 z)X + (b_0 + b_1 z), \quad (7)$$

i.e., we are Taylor expanding to the first order the unknown dependence of the slope and zeropoint on the redshift. As a general result, we find that the best fit parameters and the median values of the evolutionary coefficients  $(\log a_1, \log b_1)$  are typically quite small indicating that the dependence of both the slope and the zeropoint on the redshift is quite weak, if present at all.

## 5. GRBs Hubble diagram

Once the calibration parameters for a given  $Y - X$  correlation have been obtained, it is then possible to estimate the distance modulus of

a given GRB from the measured value of  $X$ , as shown in Eqs.(2), where  $(a, b)$  are the best fit coefficients for the given  $Y - X$  correlation, while  $\kappa = 4\pi P_{bolo}$ ,  $\kappa = 4\pi S_{bolo} F_{beam}/(1+z)$  and  $\kappa = 4\pi S_{bolo}/(1+z)$  for  $Y = L$ ,  $Y = E_\gamma$  and  $E_{iso}$ , respectively. It is then possible to both reduce the uncertainties and (partially) wash out the hidden systematic errors by averaging over the different correlations available for a given GRB.

### 5.1. Impact of the calibration method

Fig. 1 shows the GRBs Hubble diagrams (hereafter, HDs) obtained averaging over the above 2D correlations and using the three different calibration methods. The red solid line is the expected  $\mu(z)$  curve for the fiducial  $\Lambda$ CDM model.

As a general remark, we find that, notwithstanding the calibration method adopted, the GRBs HDs reasonably follow the  $\Lambda$ CDM curve although with a non-negligible scatter. Quite surprisingly, the scatter is significantly larger in the range  $0.4 \leq z \leq 1.4$  because of a set of GRBs with  $\mu(z)$  lying systematically above the  $\Lambda$ CDM prediction. One should argue for a failure of the theoretical model, but there are actually a set of points which are hard to reconcile with any reasonable dark energy model.

In order to compare the HDs from the three different calibration methods, we consider the values of  $\Delta\mu = \mu_{fid}(z) - \mu(z)$  with  $\mu_{fid}(z)$  the theoretically predicted distance modulus for the fiducial  $\Lambda$ CDM model and then we conclude that the HDs, obtained by using different calibration methods, are consistent with each other within the uncertainties.

### 5.2. Impact of the averaging procedure

As yet stated above, averaging the  $\mu$  values from different correlations helps reducing the total uncertainties and partially washes out the systematics connected to each single scaling relations.

As a first check, we compare the  $\Delta\mu$  values obtained estimating  $\mu$  using each single cor-

relation. While the median values of  $\Delta\mu$  are roughly comparable, both  $\langle\Delta\mu\rangle$  and  $(\Delta\mu)_{rms}$  are definitely larger for the  $L - E_{peak}$  and  $L - V$  correlations. Pending the question of which relation is physical, we can quantify the impact of an incorrect assumption by evaluating again the distance moduli excluding the  $L - V$  and  $L - E_{peak}$  correlations.

### 5.3. Satellite dependence

The GRBs sample is made out by collecting the data available in the literature so that the final catalog is not homogenous at all. In order to investigate whether this could have any impact on the HD, we consider again the deviations from the fiducial  $\Lambda$ CDM model using only the 80 GRBs detected with the *Swift* satellite. Somewhat surprisingly, we find larger  $\Delta\mu$  values independent of the calibration procedure adopted.

## 6. Conclusions

GRBs have recently attracted a lot of attention as promising candidates to expand the Hubble diagram up to very high  $z$ .

As the Phillips law is the basic tool to standardize SNIa, the hunt for a similar relation to be used for GRBs has lead to different empirically motivated 2D scaling relations. However, the lack of a local GRBs sample leads to the so called *circularity problem*. In an attempt to overcome this problem, we have here considered the impact on the scaling relations and GRBs HD of three different methods to estimate the luminosity distance, concluding that they lead to consistent results. The Hubble diagrams averaging over the correlations considered is not affected by the choice of the calibration method.

Once the calibration procedure has been adopted, one has still to check whether a redshift evolution of the GRBs scaling re-

lations is present or not. We have therefore explored two different parameterizations concluding that such an evolution is not statistically motivated and it can be neglected.

Assuming that no evolution is present, we have finally checked that the derived Hubble diagrams are not affected by systematics related to the choice of the calibration method, the averaging procedure or the homogeneity of the sample. As such, the GRBs HD could be safely used as a tool to constrain cosmological parameters.

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